

Sharp Trapped-Mode Resonances in Planar Metamaterials with a Broken Structural Symmetry

V. A. Fedotov,^{1,*} M. Rose,¹ S. L. Prosvirnin,² N. Papasimakis,¹ and N. I. Zheludev^{1,†}

¹*Optoelectronics Research Centre, University of Southampton, SO17 1BJ, United Kingdom*

²*Institute of Radio Astronomy, National Academy of Sciences of Ukraine, Kharkov, 61002, Ukraine*

(Received 23 February 2007; published 2 October 2007)

We report that a resonance response with a very high quality factor can be achieved in a planar metamaterial by introducing symmetry breaking in the shape of its structural elements, which enables excitation of trapped modes, i.e., modes that are weakly coupled to free space.

DOI: [10.1103/PhysRevLett.99.147401](https://doi.org/10.1103/PhysRevLett.99.147401)

PACS numbers: 78.66.Sq, 42.25.Bs, 42.70.-a, 78.67.-n

Metamaterials research has attracted a tremendous amount of attention in recent years. The interest is mainly driven by the opportunity of achieving new electromagnetic properties, some with no analog in naturally available materials. Extraordinary transmission [1], artificial magnetism and negative refraction [2], invisible metal [3], magnetic mirror [4], asymmetric transmission [5], and cloaking [6] are just a few examples of the new phenomena that have emerged from the development of artificially structured matter.

The exotic and often dramatic physics predicted for metamaterials is underpinned by the resonant nature of their response, and therefore, achieving resonances with high-quality factors is essential in order to make metamaterials' performance efficient. However, resonance quality factors (that is the resonant frequency over width of the resonance) demonstrated by conventional metamaterials are often limited to rather small values. This comes from the fact that resonating structural elements of metamaterials are strongly coupled to free space and therefore suffer significant losses due to radiation. Furthermore, conventional metamaterials are often composed of subwavelength particles that are simply unable to provide large-volume confinement of electromagnetic field necessary to support high- Q resonances. As recent theoretical analysis showed, high- Q resonances involving trapped (or closed) modes are nevertheless possible in metamaterials, if certain small asymmetries are introduced in the shape of their structural elements [7].

In this Letter, we report observation of exceptionally narrow transmission and reflection resonances in planar metamaterials (also known as metafilms [8] or frequency selective surfaces [9]) with weakly asymmetric structural elements. The appearance of the narrow resonances is attributed to the excitation of, otherwise inaccessible, symmetric current modes ("trapped modes"), through weak free-space coupling, which is provided by the structural symmetry breaking.

Metamaterials that were used in our experiments consisted of identical subwavelength metallic "inclusions" structured in the form of asymmetrically split rings (ASR), which were arranged in a periodic array and placed on a thin dielectric substrate (see Fig. 1). ASR patterns

were etched from 35 μm copper cladding covering IS620 PCB substrate of 1.5 mm thickness. Each copper split ring had the radius of 6 mm and width of 0.8 mm and occupied a square translation cell of 15×15 mm (see Fig. 1). Such periodic structure does not diffract normally incident electromagnetic radiation for frequencies lower than 20 GHz. The overall size of the samples used were approximately 220×220 mm. Transmission and reflection of a single sheet of this metamaterial were measured in an anechoic chamber under normal incidence conditions using broadband horn antennas.

We studied structures with two different types of asymmetry designated as type A and B in Fig. 1. The rings of type A had two equal splits dividing them into pairs of arcs of different length corresponding to 140 and 160 deg [see Fig. 1(a)]. The rings of type B were split along their

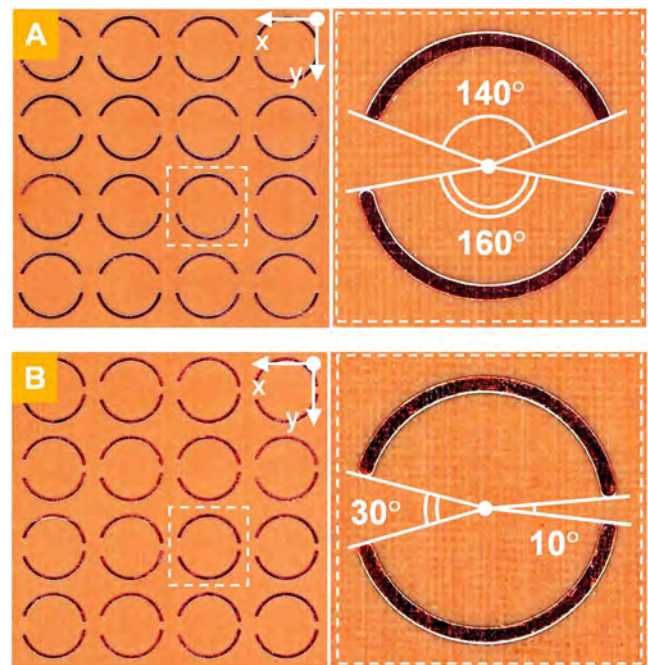


FIG. 1 (color online). Fragments of planar metamaterials with asymmetrically split copper rings. The dashed boxes indicate elementary translation cells of the structures.

diameter into two equal parts but had splits of different length corresponding to 10 and 30 deg [see Fig. 1(b)].

Transmission and reflection properties of structures of both types depended strongly on the polarization state of incident electromagnetic waves. The most dramatic spectral selectivity was observed for the electrical field being perpendicular to the mirror line of the asymmetrically split rings, which corresponded to x -polarization in the case of structure *A* and y -polarization for structure *B* (as defined in Fig. 1). For the orthogonal polarizations, the ASR structures did not show any spectral features originating from asymmetrical structuring.

The results of reflection and transmission measurements of metamaterial *A* obtained for x -polarization are presented in Fig. 2(a). The reflection spectrum reveals an ultrasharp resonance near 6 GHz (marked as II), where reflectivity losses exceed -10 dB. It is accompanied by two much weaker resonances (marked as I and III) corresponding to reflection peaks at about 5.5 and 7.0 GHz, respectively. The sharp spectral response in reflection is matched by a very narrow transmission peak reaching -3 dB and having the width of only 0.27 GHz as measured at 3 dB below the maximum. The quality factor Q of such response is 20, which is larger than that of the most metamaterials based on lossy PCB substrates by at least 1 order of magnitude. On both sides of the peak, the transmission decreases resonantly to about -35 dB at frequencies corresponding to reflection maxima.

Figure 3(a) presents transmission and reflection spectra of *B*-type metamaterial measured for y -polarization. A very narrow resonant transmission dip can be seen near 5.5 GHz, where transmission drops to about -5 dB. The corresponding reflection spectrum shows an unusually sharp roll-off (I-II) between -4 and -14 dB, spanning only 0.13 GHz at around the same frequency. At the frequency of about 11.5 GHz, the ASR structure exhibits its fundamental dipole reflection resonance (marked as III), where the wavelength of excitation becomes equal to the length of the arcs.

To understand the resonant nature of the response, the ASR structures were modeled using the method of moments. It is a well established numerical method, which involves solving the integral equation for the surface current induced in the metal pattern by the field of the incident wave. This is followed by calculations of scattered fields as a superposition of partial spatial waves. The metal pattern is treated as a perfect conductor, while the substrate is assumed to be a lossy dielectric. For both transmission and reflection, the theoretical calculations show a very good agreement with the experimental results assuming $\epsilon = 4.07 + i0.05$ (see Figs. 2 and 3, filled circles). For comparison, we also modeled metamaterial composed of split rings with no structural asymmetry, i.e., equally split along their diameter. Our calculations indicate that for both polarizations, the response of such structure is free from sharp high- Q resonant feature (see Figs. 2 and 3, open circles).

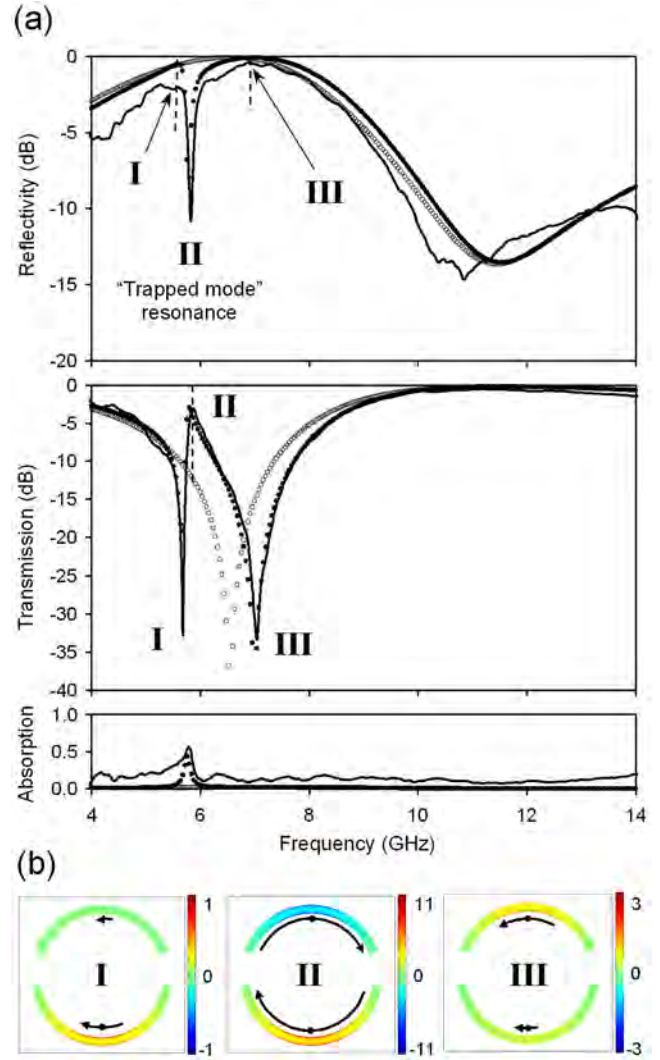


FIG. 2 (color online). (a) Normal incidence reflection and transmission and absorption (linear scale) spectra of *A*-type metamaterial [presented in Fig. 1(a)] for x -polarization: solid line—experiment, filled circles—theory (method of moments), empty circles—theory for reference structure with symmetrically split rings. (b) x -Component of the instantaneous current distribution in the asymmetrically split rings corresponding to resonant features I, II, and III marked in section (a). Arrows indicate instantaneous directions of the current flow, while their length corresponds to the overall current strength, and the midpoint shows location of the current density maximum.

The origin of the unusually strong and narrow spectral responses of the ASR structures can be traced to so-called “trapped modes,” i.e., electromagnetic modes that are weakly coupled to free space. It is this property of the trapped modes that allows in principal to achieve high-quality resonances in very thin structures [7]. These modes are normally inaccessible in symmetrically split rings, but can be excited if the metamaterial’s particles have certain structural asymmetry that allows weak coupling to free space.

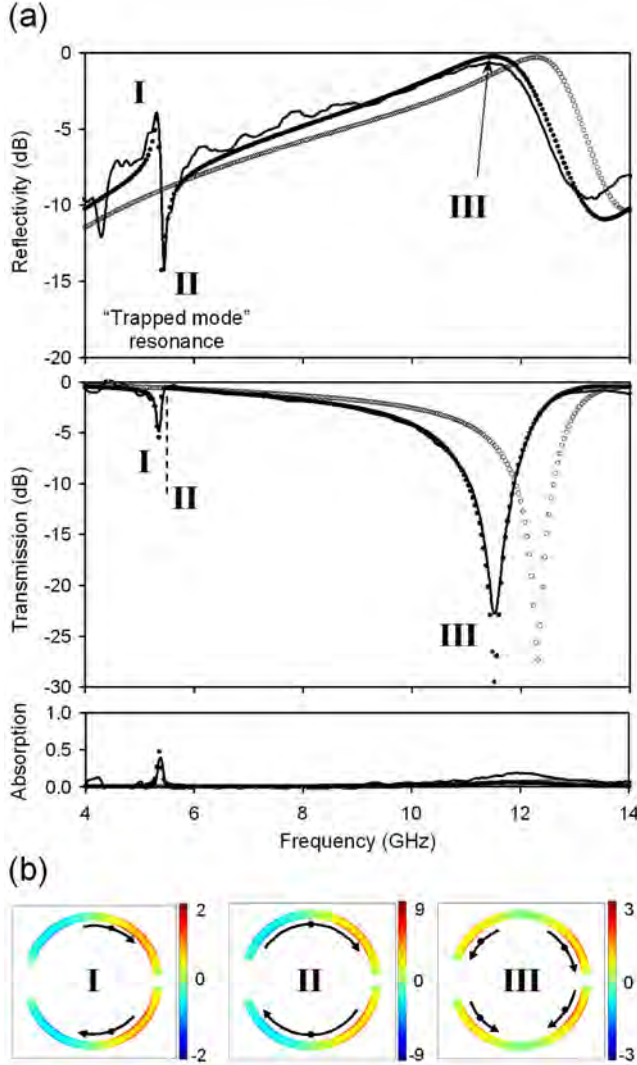


FIG. 3 (color online). (a) Normal incidence reflection and transmission, and absorption (linear scale) spectra of *B*-type metamaterial [presented in Fig. 1(b)] for *y*-polarization: solid line—experiment, filled circles—theory (method of moments), empty circles—theory for reference structure with symmetrically split rings. (b) *y*-Component of the instantaneous current distribution in the asymmetrically split rings corresponding to resonant features I, II, and III marked in section (a). Arrows indicate instantaneous directions of the current flow, while their length corresponds to the current strength and the midpoint shows location of the current density maximum.

Our calculations showed that in the case of structure A, an antisymmetric current mode can dominate the usual symmetric one (dipole mode): at the high-*Q* transmission resonance, as shown in Fig. 2(b) (II), two parts of the ring are excited in antiphase, while currents have almost the same amplitude. The scattered electromagnetic fields produced by such current configurations are very weak, which dramatically reduces coupling to free space and therefore radiation losses. Indeed, both the electric and magnetic dipole radiation of currents oscillating in the arcs of the neighboring rings is cancelled. As a consequence, the

strength of the induced currents can reach very high values and therefore ensure a high-quality factor of the response. At the dipole reflection resonances, in contrast to the “trapped-mode” regime, currents in both sections of the asymmetrically split ring oscillate in phase, but excitation of one of the sections dominates the other [see Figs. 2(b) (I and III)]. Importantly, the amplitudes of the currents in this case are significantly smaller than at trapped-mode resonance despite lower dissipation losses, which yields lower *Q*-factors for this type of the response.

Our numerical analysis showed that, with the reduction of particle structural asymmetry, all resonances including the “trapped-mode” resonance shift towards lower frequencies (see Fig. 4). In the lossless case, the magnitude of the resonance depends weakly on the asymmetry, while its quality factor monotonically increases. In the presence of losses, the quality factor at first increases with decreasing asymmetry, peaking at $\gamma \approx 0.09$. As γ decreases further, the role of the dissipative losses starts to increase and the “trapped-mode” resonance becomes weaker and broader until it disappears in a fully symmetric particle.

Interpretation of the results obtained for structure *B* appears to be slightly more elaborate. From the symmetry of the split rings, it follows that for *y*-polarized excitation at any frequency, current distribution in the opposite sections of the ring should have equal *y*-components oscillating in phase and equal *x*-components oscillating in antiphase. The net current in the ring therefore always has zero *x*-component, while its *y*-component cannot be fully compensated due to the structural asymmetry. At low frequencies, the net *y*-component is small, but it increases significantly as the frequency of excitation approaches 5.5 GHz. At this frequency, the wavelength becomes equal to the circumference of the split ring and, as shown in Fig. 3(b) (I), the right side of the ring dominates its left side oscillating in antiphase. The later results in a dipolelike

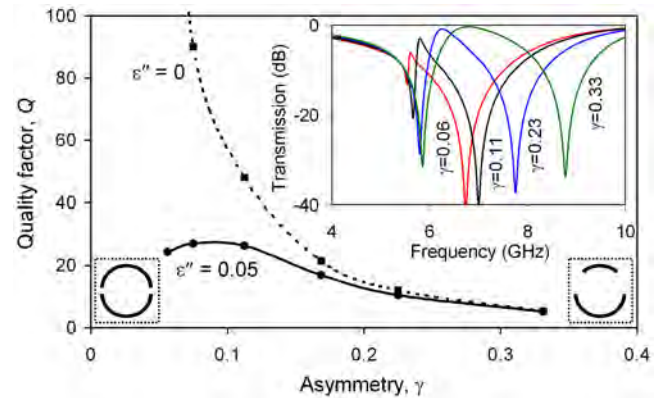


FIG. 4 (color online). Quality factor of “trapped-mode” resonance in *A*-type metamaterial as a function of the particle asymmetry γ , defined as the relative difference of the arcs’ lengths. Solid and dashed curves correspond to the cases of lossy ($\epsilon = 4.07 + i0.05$) and lossless ($\epsilon = 4.07$) substrate. The inset shows the evolution of “trapped-mode” resonance in transmission at different γ calculated for the lossy case.

resonant increase of the metamaterial reflection [see Fig. 3(a)]. Immediately above this frequency, resonance contributions of both sides of the ASR-particle are still in antiphase but become nearly identical [see Fig. 3(b) (II)]. As in the case of A-type metamaterial, both electric and magnetic dipole radiation of such current configuration is cancelled for neighboring particles, thus dramatically reducing radiation losses of the metamaterial (reflection). Further increase of the excitation frequency leads to a rise of the reflection until the fundamental dipole resonance of the ASR structure is reached where the corresponding wavelength is equal to the length of the arcs. In this case, both sides of the ASR particle oscillate in phase and equally contribute to electromagnetic field scattering [see Fig. 3(b) (III)].

The quality factor of the trapped-mode resonances will increase on reducing the degree of asymmetry of metamaterial particles and, in the case of low dissipative losses, can be made exceptionally high (see Fig. 4). In the microwave region, metals are almost perfect conductors, and the main source of dissipative losses is the substrate material (dielectrics). In the present case, for example, absorption in the substrate at the trapped-mode resonance reaches about 50% for both types of the metamaterial [see Fig. 2(a) and 3(a)]. Significantly higher resonance quality factors, therefore, can be achieved for freestanding thin metal films, which are patterned complimentary to ASR structures, i.e., periodically perforated with ASR openings. In the visible and IR spectral ranges, however, losses in metals dominate, and therefore nanoscaled versions of the original metal-dielectric ASR structures would perform better. According to our estimates, the Q -factor of such ASR nanostructures in the near-IR can be as high as six.

It is interesting to note that the symmetry-breaking trapped-mode resonances in planar metamaterials appear to belong to a large family of characteristically asymmetric Fano resonances, resulting from the interference between a high-quality resonance and a much smoother, continuum-like spectrum [10]. In the past, Fano-type resonances were observed in various systems including micro pillars, whose high-quality whispering-mode optical resonances interfered through evanescent coupling with a broad TIR resonance of a prism [11], and in artificial composites, such as sonic [12] and photonic [13] crystals. Intriguingly, trapped-mode resonances here also seem to be another classical analog of narrow resonances observed in electromagnetically induced transparency [14].

In summary, we experimentally and theoretically showed that a new type of planar metamaterials (metafilms) composed of asymmetrically split rings exhibit unusually strong high- Q resonances and provide for extremely narrow transmission and reflection pass and stop bands. The metamaterials' response has a quality factor of about 20, which is one order of magnitude larger than the typical value for many conventional metamaterials or metafilms. This is achieved via weak coupling between "trapped modes" in the resonant inclusions of the ASR

metamaterial and free space, while weak symmetry breaking enables excitation of so-called "trapped modes." Achieving the "trapped-mode" resonances will be especially important for artificial structures in the optical part of the spectrum, where losses are significant and unavoidable. In a certain way, such symmetry-breaking resonances in metamaterials resemble the recently identified spectral lines of plasmon absorption of shell nanoparticles appearing due to asymmetry [15] and "forbidden" chiral response in second-harmonic generation by L-shaped nanoparticles due to symmetry-breaking defects [16].

The authors would like to acknowledge the financial support of the EPSRC (UK) and Metamorphose NoE.

*n.i.zheludev@soton.ac.uk

†www.nanophotonics.org.uk

- [1] W. L. Barnes, A. Dereux, and T. W. Ebbesen, *Nature* (London) **424**, 824 (2003).
- [2] D. R. Smith, J. B. Pendry, and M. C. K. Wiltshire, *Science* **305**, 788 (2004).
- [3] V. A. Fedotov, P. L. Mladyonov, S. L. Prosvirnin, and N. I. Zheludev, *Phys. Rev. E* **72**, 056613 (2005).
- [4] A. S. Schwanecke, V. A. Fedotov, V. Khardikov, S. L. Prosvirnin, Y. Chen, and N. I. Zheludev, *J. Opt. A Pure Appl. Opt.* **9**, L1 (2007).
- [5] V. A. Fedotov, P. L. Mladyonov, S. L. Prosvirnin, A. V. Rogacheva, Y. Chen, and N. I. Zheludev, *Phys. Rev. Lett.* **97**, 167401 (2006).
- [6] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, *Science* **314**, 977 (2006).
- [7] S. Prosvirnin and S. Zouhdi, in *Advances in Electromagnetics of Complex Media and Metamaterials*, edited by S. Zouhdi *et al.* (Kluwer Academic Publishers, The Netherlands, 2003), pp. 281–290.
- [8] E. Kuester, M. Mohamed, M. Piket-May, and C. Holloway, *IEEE Trans. Antennas Propag.* **51**, 2641 (2003).
- [9] J. Vardaxoglou, *Frequency Selective Surfaces* (Research Studies Press LTD., England, 1997).
- [10] U. Fano, *Phys. Rev.* **124**, 1866 (1961).
- [11] H. T. Lee and A. W. Poon, *Opt. Lett.* **29**, 5 (2004).
- [12] C. Goffaux, J. Sanchez-Dehesa, A. L. Yeyati, P. Lambin, A. Khelif, J. O. Vasseur, and B. Djafari-Rouhani, *Phys. Rev. Lett.* **88**, 225502 (2002).
- [13] S. Fan, W. Suh, and J. D. Joannopoulos, *J. Opt. Soc. Am. A* **20**, 569 (2003).
- [14] R. Boyd and D. Gauthier, *Nature* (London) **441**, 701 (2006).
- [15] H. Wang, Y. Wu, B. Lassiter, C. L. Nehl, J. H. Hafner, P. Nordlander, and N. J. Halas, in *Proceedings of the National Academy of Science of the United States of America* (Natl. Acad. Sciences, Washington, DC, 2006), Vol. 103, p. 10856.
- [16] B. K. Canfield, S. Kujala, K. Laiho, K. Jefimovs, J. Turunen, and M. Kauranen, *Opt. Express* **14**, 950 (2006).